Seminary 1 Electric charge, force, field, potential

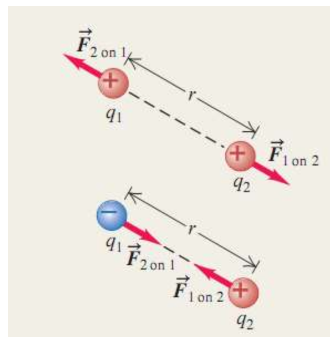
The unsolved problems are given as homework.

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

Coulomb's law: For charges q1, q2 and separated by a distance *r* => force

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

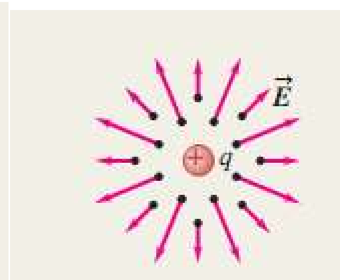
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}$$



Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge.

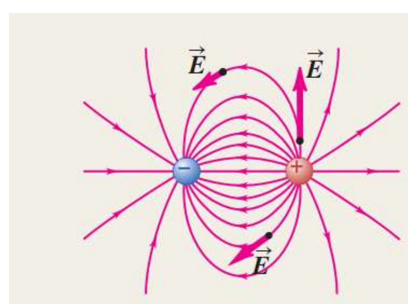
$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$



Superposition of electric fields: The electric field $\vec{E}$ in a point $\vec{r}$, of any combination of sources (charges) is the vector sum of the field caused by each individual charges: $\vec{E}(\vec{r}) = \sum_i \vec{E}_i(\vec{r}_i\ )$.

Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.



Problems:

1/ What similarities do electrical forces have with gravitational forces? What are the most significant differences?

Similitudes:

Same square mathematical law. The force between two electric charges is proportional to the product of the two charges divided by the distance between them squared, just as the force between two masses is proportional to the product of the two masses divided by the distance between them squared.

Both are conservative. Derive from a conservative field (work does not depend on path). For both one can define a potential energy.

Differences

1/ the electrical force is much stronger than the gravitational force (give example: charged balloon stick to wall, etc..).

2/ the gravitational force is always attractive. The electrical force can be either attractive or repulsive. There is only one type of mass, but there are two types of electric charge. Like charges will repel each other and unlike charges will attract.

2/ Calculate the ratio between electrostatic force and gravitational force

- **between two electrons**
- **between two protons.**

Atomic nuclei are made of protons and neutrons. This shows that there must be another kind of interaction in addition to gravitational and electric forces. Explain.

Answer

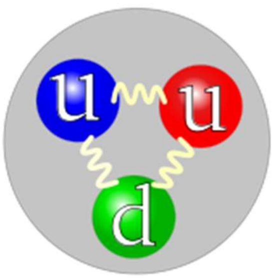
The electric force is given by Coulomb's Law, while the gravitational force is given by Newton's Law of Universal Gravitation.

- m = the mass of an electron = 9.1×10^{-31} kg
- q = the electric charge on an electron = 1.6×10^{-19} C
- r = the distance between the two electrons
- G = Universal Gravitation Constant = 6.67×10^{-11} N m^2/kg^2
- k = 8.9×10^9 N m^2/C^2

$$\frac{F_{elec}}{F_{grav}} = \frac{k\frac{q^2}{r^2}}{G\frac{m^2}{r^2}} = \frac{kq^2}{Gm^2} = \frac{\left(8.9 x 10^9 \frac{Nm^2}{C^2}\right)\left(1.6 x 10^{-19} C\right)^2}{\left(6.7 x 10^{-11} \frac{Nm^2}{kg^2}\right)\left(9.1 x 10^{-31} kg\right)^2} = 4.1 x 10^{42}$$

The electric force between two electrons is always 4.1 x 10^{42} times as great as the gravitational force between them *at any distance!*

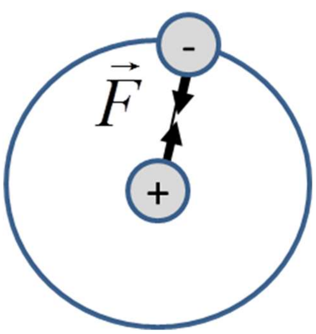
The mass of the proton is 1.7 x 10^{-27}kg. that the ratio of F_{elec}/F_{grav} is about 1.2 x 10^{36} for protons, regardless of the distance between them. Therefore, the gravitational force is certainly ***not*** the force that holds protons together in the nucleus of the atom! Protons and neutrons are held together in a nucleus of an atom by the strong force (strongest attractive force, one of the 4 fundamental forces (strong, weak, electromagnetic and gravity). According to the standard model of particle physics, the fundamental forces (strong, weak, electromagnetic and gravity) are predicted to occur as a result of an exchange between particles via "force carrying particles". Also, neutrons and protons are made up of tinier particles called **quarks**. And it is the quarks that exchange force carrying particles between each other to give rise to the strong force.



6 different quarks, classified by flavours: up, down, charm, strange, top, bottom
Proton =(uud), neutron (u,d,d).
The force carrying particles are called gluons. The strong force only operates at EXTREMELY small distances (of order 10 $^{-15}$m). The strong force also attracts protons to protons or neutrons to neutrons.

3/ According to Bohr's theory, in the Hydrogen atom the electron rotates around the proton along a circular orbit with a radius of 0.529 10^{-10}m.

- **a) Calculate the electric force between the two particles.**
- **b) Calculate the electron speed considering the electric force as centripetal.**



$$F = \frac{e|e|}{4\pi\varepsilon_0 r^2} = 9\cdot10^9\frac{e^2}{r^2} = 9\cdot10^9\frac{\left(1.6\cdot10^{-19}\right)^2}{\left(0.529\cdot10^{-10}\right)^2}$$

$$F = 8.22\cdot10^{-8}N$$

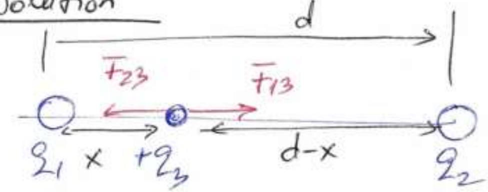
The force being centripetal =>

$$F = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.22\cdot10^{-8}\cdot0.529\cdot10^{-10}}{9.1\cdot10^{-31}}} = 2.19\cdot10^6 m/s$$

!!!

4/ Two electrical charges, q1=+3q and q2=+q are separated by the distance d. At what distance x, has to be placed a 3rd charge q3 in order to be in equilibrium?

Solution



equilibrium means: $\sum \vec{F_i} = 0 \Rightarrow F_{13} = F_{23}$

$$\frac{q_1 q_3}{4\pi\varepsilon_0 x^2} = \frac{q_2 q_3}{4\pi\varepsilon_0 (d-x)^2} \Rightarrow \text{2nd order eq.}$$

$$\boxed{x^2(q_1 - q_2) - 2dq_1 x + d^2 q_1 = 0}$$

$q_1 = 3q \Rightarrow q_2 = q$

$$2x^2 - 6dx + 3d^2 = 0$$

Solution

$$x_{1,2} = \frac{-B \pm \sqrt{\Delta}}{2A} = \frac{6d \pm \sqrt{36d^2 - 24d^2}}{4} = \left(\frac{3 \pm \sqrt{3}}{2}\right)d$$

The only valid solution which lies between q_1 and q_2 is

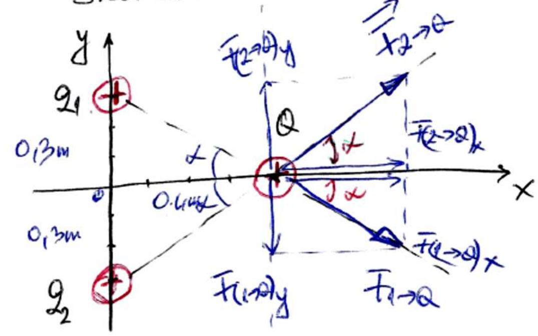
$$x = \frac{3-\sqrt{3}}{2} d = 0{,}6d$$

(the initial equilibrium condition has been written so that $0 < x < d$)

5/ Two equal positive charges q1=q2=2μC are located at x=0, y=0.3 and x=0, y=-0.3m, respectively. What are the magnitude and the direction of the total electric force that q1 and q2 exert on a third charge q=4μC at x=0.4, y=0.

Solution

Sketch:



we will use the components of forces to calculate the VECTOR SUM

$$F_{1\to Q} = |\vec{F}_{1\to Q}| = \frac{q_1 Q}{4\pi\varepsilon_0 (r_{1\to Q})^2} \qquad q_1 = q_2 \Rightarrow F_{1Q} = F_{2Q}$$

$$F_{2\to Q} = |\vec{F}_{2\to Q}| = \frac{q_2 Q}{4\pi\varepsilon_0 (r_{2\to Q})^2}$$

$$r_{1\to Q} = r_{2\to Q} = \sqrt{0{,}3^2 + 0.4^2} = \underline{0{,}5\,m}$$

$F_{(1\to Q)y} = F_{1\to Q}\sin\alpha$ is equal and opposite (see (y))

to $F_{(2\to Q)y} = F_{2\to Q}\sin\alpha \Rightarrow \underline{\sum F_y = 0}$

$$F_{(1\to Q)x} = F_{(2\to Q)x} = F_{1\to Q}\cos\alpha = \frac{q_1 Q}{4\pi\varepsilon_0 (r_{1\to Q})^2}\cos\alpha$$

$\cos\alpha = \frac{0.4}{0{,}5}$ (from geometrical construction)

$$F_{1\to Q} = \frac{q_1 Q}{4\pi\varepsilon_0 (r_{1\to Q})^2} = \frac{(4\cdot 10^{-6}C)(2\cdot 10^{-6}C)}{(0{,}5)^2\,m^2}\, 9\cdot 10^9 \frac{Nm^2}{C^2}$$

$$= 0{,}29\,N$$

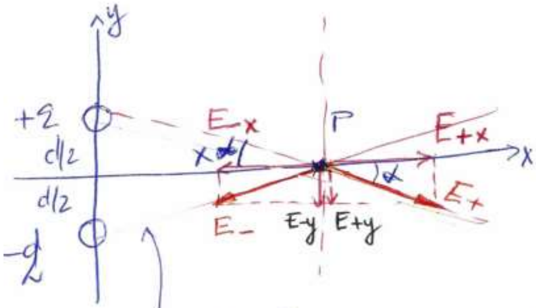
$$F_x = 2F_{1\to 2}\cdot\cos\alpha = 2\cdot 0{,}29\cdot\frac{0.4}{0.5}\,N = \underline{0{,}46\,N}$$

The total force is along x direction and has a magnitude of 0,46 N.

ELECTRIC FIELD AND SUPERPOSITION

Electric dipole

Consider an electric dipole composed by two equal charges and opposite sign +q and –q separated by the distance d. Calculate the electric field of an arbitrary point in the median plane, at a distance x>>d.



$$E_+ = E_- = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)}$$

The x component cancels reciprocally

$$r^2 = x^2 + \frac{d^2}{4}$$

on y

$$E_{+y} = E_{-y} = E_+\sin\alpha$$

$$\sin\alpha = \frac{d/2}{\sqrt{x^2+\frac{d^2}{4}}} \qquad \Rightarrow$$

$$E_y = E_{+y} + E_{-y} = 2E_{+y}$$

$$E_y = 2\cdot\frac{q}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)}\cdot\frac{d/2}{\sqrt{x^2+\frac{d^2}{4}}}$$

$$= \frac{qd}{4\pi\varepsilon_0}\,\frac{1}{\left(x^2+\frac{d^2}{4}\right)^{3/2}} = \frac{p}{4\pi\varepsilon_0 x^3}\,\frac{1}{\left(1+\frac{d^2}{4x^2}\right)^{3/2}}$$

$$= \frac{p}{4\pi\varepsilon_0 x^3}\left(1+\frac{d^2}{4x^2}\right)^{-3/2}$$

Mclaurin for $x \gg d \Leftrightarrow \frac{d^2}{4x^2} \ll 1$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$
$$= \sum_{n\geq 0} \frac{f^{(n)}(0)}{n!} \cdot x^n$$

$$f(x) = (1+x)^n \qquad f'(x) = n(1+x)^{n-1}$$
$$f(0) = 1 \qquad f'(0) = n$$
$$\Rightarrow (1+x)^n \simeq 1 + nx$$

$$(1+y)^n \simeq 1 + ny$$

here $y = \frac{d^2}{4x^2}$

$n = -3/2$

$$\Rightarrow \left(1 + \frac{d^2}{4x^2}\right)^{-3/2} \triangleq 1 - \frac{3}{2}\frac{d^2}{4x^2}$$

$$\Rightarrow E_y \triangleq \frac{p}{4\pi\varepsilon_0 x^3}\left(1 - \frac{3}{2}\frac{d^2}{4x^2}\right)$$

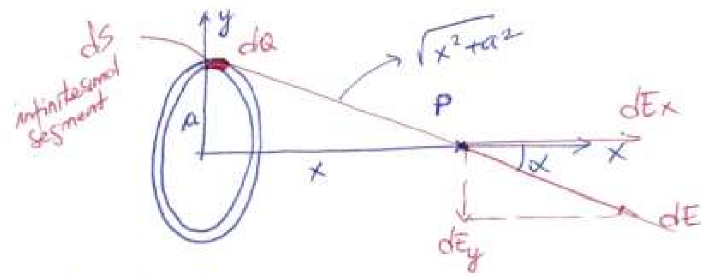
$$\boxed{E_y \simeq \frac{p}{4\pi\varepsilon_0 x^3}}$$

we neglect the 2nd term in $\frac{d^2}{x^6}$

Field of a ring of charge **Charge is uniformly distributed around a conducting ring of radius a (Fig.). Find the electric field at a point P on the ring axis at a distance from its center.**

Solution

Each bit of charge around the ring produces an electric field at an arbitrary point on the x axis.



$Q = \lambda \cdot 2\pi a$

$dq = \lambda dS$

From symmetry reasons $\sum dE_y = 0$

$$dE_x = dE \cos\alpha = dE \frac{x}{\sqrt{a^2+x^2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{dq}{4\pi\varepsilon_0 (a^2+x^2)} \quad \Rightarrow$$

$$dE_x = \frac{\lambda \, dS \, x}{4\pi\varepsilon_0 (a^2+x^2)^{3/2}}$$

to get total E_x we integrate this expression over the ring that is S from 0 to $2\pi a$

$$E_x = \int dE_x = \int_0^{2\pi a} \frac{\lambda \, dS \, x}{4\pi\varepsilon_0 (a^2+x^2)^{3/2}}$$

$$E_x = \frac{\lambda x}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}} \int_0^{2\pi a} dS$$

$$\Rightarrow \boxed{E_x = \frac{1}{4\pi\varepsilon_0} \frac{\lambda x}{(x^2+a^2)^{3/2}} (2\pi a)} \qquad \lambda = \frac{Q}{2\pi a}$$

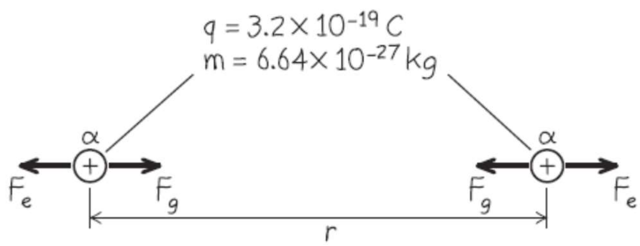
$$\boxed{E_x = \frac{1}{4\pi\varepsilon_0} \frac{Q\,x}{(x^2+a^2)^{3/2}}}$$

Homework

1/ *Two metal spheres are hanging from nylon threads. When you bring the spheres close to each other, they tend to attract. Based on this information alone, discuss all the possible ways that the spheres could be charged. Is it possible that after the spheres touch, they will cling together? Explain.*

2/ *An uncharged metal sphere hangs from a nylon thread. When a positively charged glass rod is brought close to the metal sphere, the sphere is drawn toward the rod. But if the sphere touches the rod, it suddenly flies away from the rod. Explain why the sphere is first attracted and then repelled.*

3/ *An α particle (the nucleus of a helium atom) has mass $m=6.64\ 10^{-27}$kg and charge $q=+2e=3.2\ 10^{-19}$C. Compare the magnitude of the electric repulsion between two ("alpha") particles with that of the gravitational attraction between them.*



4/ *The free electrons in a metal are gravitationally attracted toward the earth. Why, then, don't they all settle to the bottom of the conductor, like sediment settling to the bottom of a river?*

5/ *A proton is placed in a uniform electric field and then released. Then an electron is placed at this same point and released. Do these two particles experience the same force? The same acceleration? Do they move in the same direction when released?*